
ANALYZIS OF MATRIX EIGENVALUES AND EIGENVECTORS

(diary output of a MATLAB session using "MATRIX-0.M").

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```
clearwhos

matrix_0; [ or "matrix-0"]
```

LIST OF NAMES AND SIZES OF MATRICES TO BE ANALYZED

whos

Name	Size	Elements	Bytes	Density	Complex
GK3	3 by 3	9	72	Full	No
GK3_anti	3 by 3	9	72	Full	No
GK3_sym	3 by 3	9	72	Full	No
I1	1 by 1	1	8	Full	No
I2	2 by 2	4	32	Full	No
I3	3 by 3	9	72	Full	No
J2	2 by 2	4	32	Full	No
J3	3 by 3	9	72	Full	No
K2	2 by 2	4	32	Full	No
K3	3 by 3	9	72	Full	No
KK2	2 by 2	4	32	Full	No
KK3	3 by 3	9	72	Full	No
LAP2	2 by 2	4	32	Full	No
LAP3	3 by 3	9	72	Full	No
LAP4	4 by 4	16	128	Full	No

Grand total is 109 elements using 872 bytes

Analyzis of matrices GK3, Sym(GK3), and AntiSym(GK3)

♦ GK3

MATRIX "GK3"

GK3 =

+8	-1	-5
-4	+4	-2
+18	-5	-7

➢ [V,D]=eig(GK3)

```
V =
  -0.0651 - 0.4424i  -0.0651 + 0.4424i   0.4082       Columns are
   0.3773 - 0.5076i   0.3773 + 0.5076i   0.8165       EIGENVECTORS
  -0.5076 - 0.3773i  -0.5076 + 0.3773i   0.4082       of GK3

D =
   2.0000 + 4.0000i        0                  0       Diagonal terms are
        0             2.0000 - 4.0000i        0       EIGENVALUES
        0                  0             1.0000       of GK3
```

➢ [V,D]=eig(GK3_sym)

```
V =
   -0.4522   -0.8347   -0.3143            Columns are
   -0.8917    0.4158    0.1787            EIGENVECTORS
    0.0185   -0.3611    0.9324            of Sym(GK3)

D =
    2.8047         0         0            Diagonal terms are
         0   12.0572         0            EIGENVALUES
         0         0   -9.8619            of Sym(GK3)
```

➢ [V,D]=eig(GK3_anti)

```
V =
  -0.7013 - 0.0000i  -0.7013 + 0.0000i   0.1283    Columns are
   0.0899 - 0.0915i   0.0899 + 0.0915i   0.9834    EIGENVECTORS
   0.0117 + 0.7012i   0.0117 - 0.7012i   0.1283    of AntiSym(GK3)

D =
   0.0000 +11.6940i        0                  0    Diagonal terms are
        0             0.0000 -11.6940i        0    EIGENVALUES
        0                  0             0.0000    of AntiSym(GK3)
```

```
-----------------------------------
Analyzis of identity matrix I3
-----------------------------------
```

$\boxed{\text{I3}}$

MATRIX "I3"

I3 =

+1	0	0
0	+1	0
0	0	+1

```
[V,D]=eig(I3)

V =
     1     0     0
     0     1     0          EIGENVECTORS of I3
     0     0     1

D =
     1     0     0
     0     1     0          EIGENVALUES of I3
     0     0     1
```

```
-------------------------------------------------
Analyzis of two special anti-symmetric matrices
-------------------------------------------------
```

♦ J2

```
J2 =            MATRIX "J2"

                  0    +1
                 -1     0

[V,D]=eig(J2)

V =
        0.7071                   0.7071
        0      + 0.7071i         0      - 0.7071i  EIGENVECTORS of J2

D =
       0 + 1.0000i        0
       0                  0 - 1.0000i            EIGENVALUES of J2
```

```
--------------------------------------------------------
```

♦ J3

```
J3 =              MATRIX "J3"

                  0      0      1
                  0      0      0
                 -1      0      0

[V,D]=eig(J3)

V =
  0.7071             0.7071                0
       0                  0           1.0000
       0 + 0.7071i        0 - 0.7071i      0

D =
       0 + 1.0000i        0                0
       0                  0 - 1.0000i      0
       0                  0                0
```

Indefinite matrices (neither positive nor negative)
Matrices indéfinies (ni positives ni négatives)

♦ **K2**

```
K2 =          MATRIX "K2"

               +1      0
                0     -1

[V,D]=eig(K2)

        V =
             1      0
             0      1

        D =
             1      0
             0     -1
```

♦ **K3**

```
K3 =          MATRIX "K3"

              1       0       0
              0      -2       0
              0       0       1

[V,D]=eig(K3)

     V =
          1      0      0
          0      1      0
          0      0      1

     D =
          1      0      0
          0     -2      0
          0      0      1
```

♦ **KK2**

```
KK2 =
     2     0
     0    -1

[V,D]=eig(KK2)

V =
     1     0
     0     1
D =
     2     0
     0    -1
```

♦ **KK3**

```
KK3 =
     1     0     0
     0    -1     0
     0     0     1

[V,D]=eig(KK3)

V =
     1     0     0
     0     1     0
     0     0     1

D =
     1     0     0
     0    -1     0
     0     0     1
```

Laplacian matrices (finite differences and discrete networks)
Matrices Laplaciennes (différences finies, réseaux discrets)

♦ LAP2

LAP2 = MATRIX "LAP2"

+2	-1
-1	+2

```
[V,D]=eig(LAP2)

V =
   -0.7071   -0.7071
    0.7071   -0.7071

D =
    3.0000         0
         0    1.0000
```

♦ LAP3

LAP3 = MATRIX "LAP3"

+2	-1	0
-1	+2	-1
0	-1	+2

```
[V,D]=eig(LAP3)

V =
    0.5000    0.7071    0.5000
    0.7071    0.0000   -0.7071
    0.5000   -0.7071    0.5000

D =
    0.5858         0         0
         0    2.0000         0
         0         0    3.4142
```

♦ LAP4

LAP4 = MATRIX "LAP4"

+2	-1	0	0
-1	+2	-1	0
0	-1	+2	-1
0	0	-1	+2

```
[V,D]=eig(LAP4)

V =
 -0.6015   0.3717  -0.6015  -0.3717
 -0.3717   0.6015   0.3717   0.6015
  0.3717   0.6015   0.3717  -0.6015
  0.6015   0.3717  -0.6015   0.3717

D =
  1.3820        0         0         0
       0   0.3820         0         0
       0        0    2.6180         0
       0        0         0    3.6180
```

Perturbations non-symétriques de matrices Laplaciennes (différences finies en advection-diffusion, etc)

♦ LAP4a

```
LAP4a =
            [      2          -0.5        0          0  ;
                  -1.5         2         -0.5        0 ;
                   0          -1.5        2         -0.5 ;
                   0           0         -1.5        2    ];
LAP4a
            LAP4a =
                      2.0000   -0.5000         0         0
                     -1.5000    2.0000   -0.5000         0
                           0   -1.5000    2.0000   -0.5000
                           0         0   -1.5000    2.0000
[V,D]=eig(LAP4a)

            V =
                0.1297   -0.1297    0.1752   -0.1752
               -0.3636   -0.3636   -0.1875   -0.1875
                0.6297   -0.6297   -0.3248    0.3248
               -0.6741   -0.6741    0.9103    0.9103

            D =
                3.4013         0         0         0
                     0    0.5987         0         0
                     0         0    2.5352         0
                     0         0         0    1.4648
```

♦ ADV4

```
            …ADV4 =
                     0    0.5000         0         0
               -0.5000         0    0.5000         0
                     0   -0.5000         0    0.5000
                     0         0   -0.5000         0

[V,D]=eig(ADV4)

V =
   0.3717              0.3717              0.6015              0.6015
        0 + 0.6015i         0 - 0.6015i         0 + 0.3717i         0 - 0.3717i
  -0.6015             -0.6015              0.3717              0.3717
        0 - 0.3717i         0 + 0.3717i         0 + 0.6015i         0 - 0.6015i

D =
        0 + 0.8090i         0                   0                   0
        0                   0 - 0.8090i         0                   0
        0                   0                   0 + 0.3090i         0
        0                   0                   0                   0 - 0.3090i
```

diary off

SINGLE PAGE LISTING OF PROGRAM "MATRIX-0.M":

```
% -----------------------------------------------------------------------------------
% Program " MATRIX-0.M "
% -----------------------------------------------------------------------------------
% Demonstration of some basic matrix properties (linear algebra) using Matlab
% -----------------------------------------------------------------------------------
% Author : R.ABABOU
% Creation date: 3 Nov 1995
% Latest update: 3 Nov 1995
% -----------------------------------------------------------------------------------
% Pour obtenir les vecteurs propres V et valeurs propres D reels ou complexes d'une matrice A,
% taper ceci sur la ligne de commande MATLAB :
%          [V,D] = eig(A);
% Attention, c'est "eig" en lettres minuscules.
% Noter que V est une matrice de vecteurs propres en colonnes,
% et D est une matrice diagonale de valeurs propres (elements diagonaux).
% ------------------------------------------------------------------------------------

% --------------------------------------------
% PRE-DEFINITION DE QUELQUES MATRICES A ETUDIER
% --------------------------------------------

  I1 = eye(1);
  I2 = eye(2);
  I3 = eye(3);

  J2 = [  0 +1   ;
         -1  0 ] ;

  J3 = [  0  0 +1   ;
          0  0  0   ;
         -1  0  0 ] ;

  K2 = [ +1  0   ;
          0 -1 ] ;

  KK2= [ +2  0   ;
          0 -1 ] ;

  K3 = [ +1  0  0   ;
          0 -2  0   ;
          0  0 +1 ] ;

  KK3= [ +1  0  0   ;
          0 -1  0   ;
          0  0 +1 ] ;

  LAP2= [ +2 -1   ;
          -1 +2 ] ;

  LAP3= [ +2 -1  0   ;
          -1 +2 -1   ;
           0 -1 +2 ] ;

  LAP4= [ +2 -1  0  0   ;
          -1 +2 -1  0   ;
           0 -1 +2 -1   ;
           0  0 -1 +2 ] ;


% DIV2=...;

% GRAD2=...;

% GK3 = matrice reelle dont les valeurs propres et vecteurs propres sont complexes.
% Les valeurs propres complexes de GK3 sont:
% Lam1=(+2,+4); Lam2=(+2,-4); Lam3=(+1,0).
% Cet exemple est tiré de <<ESSL_1987>> et de : << R.T. Gregory, D.L.Karney:
% "A collection of matrices for testing computationalalgorithms",  Wiley, 1969 >>

  GK3 = [ +8  -1  -5   ;
          -4  +4  -2   ;
         +18  -5  -7 ] ;

  GK3_sym = 0.5 * ( GK3 + GK3' ) ;

  GK3_anti= 0.5 * ( GK3 - GK3' ) ;

% ---------------------------------------------------------------------------------
% FIN DE CE PROGRAMME.
% ---------------------------------------------------------------------------------
% Lancer ce programme dans MATLAB; taper " whos" pour voir la liste des noms de matrices;
% On peut etudier une matrice (A) en tapant " [V,D]=eig(A);" comme expliqué plus haut.
% ---------------------------------------------------------------------------------
```